to the power of air to disseminate crystals of a salt thrown into it in fine powder.

De Coppet has already remarked that the mass of a solution exerts some influence on its crystallization, and I have shown that the form of the vessel also has a decided effect. The effect again of different vibrations on different solutions is worth trying, as there seems to be no reason why the hyperacid sodium salt should be an exceptional case.

A good deal of work has yet to be done before we arrive at a satisfactory explanation of these obscure phenomena.

IX. "On some Elementary Principles in Animal Mechanics.—No. VIII. The Law of Fatigue." By the Rev. SAMUEL HAUGHTON, M.D. (Dubl.), D.C.L. (Oxon.), F.R.S., Fellow of Trinity College, Dublin.

In my last paper (No. VII.) I illustrated the Law of Fatigue by experiments made in lifting weights varying in amount, without rest, at a fixed rate of motion; I shall now illustrate the Law by experiments made in lifting a fixed weight at varying rates of motion, without rest, as before.

Law of Fatigue.

"*When the same muscle (or group of muscles) is kept in constant action until fatigue sets in, the total work done multiplied by the rate of work is constant.*"

The following experiments were made during the last six months by Dr. Macalister and myself:—

A pair of 10-lb. dumbbells, held one in each hand, were raised simultaneously from the vertical to the horizontal position, and again lowered, at a rate regulated by a metronome made for the purpose. No rest was allowed at the beginning or end of the motion, which took place as before, under the following conditions, viz.:—

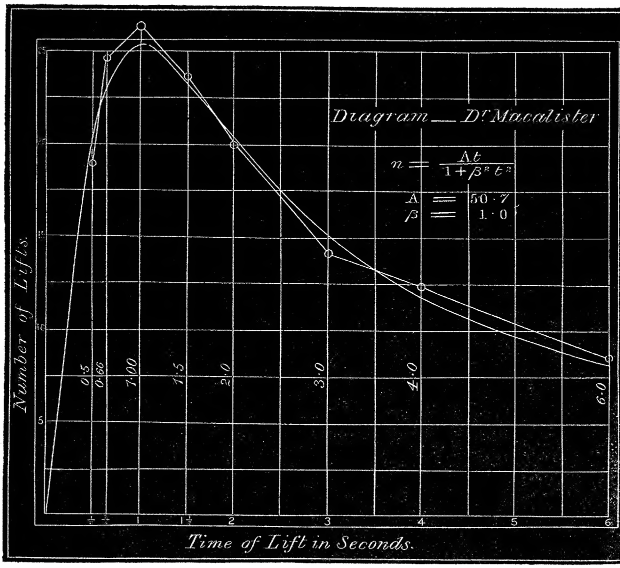
1. To keep time with the metronome.
2. To raise the weights in the transverse plane.
3. To supinate the hands.
4. To abstain from all bending of the knees or spinal column.
5. The experimenter not to count the lifts.

The experiments were made at intervals never less than 24 hours, so as to avoid all risk of the muscles becoming trained; and on each occasion the weights were lifted until it became impossible to effect another lift, without violating condition No. 4, indicating that other muscles were called in, to aid the shoulder-muscles already worn out. The following results were obtained, the exact weight of each dumbbell being 9·75 lbs.

Dr. Macalister.

Time of Lift.	Number of Lifts.										Mean.
0·50 sec.......	20	18	18	20	19	...	...	...	...	...	19·0
0·66 „	24	25	25	24	28	23	22	25	25	25	24·6
1·00 „	23	25	25	25	26	29	25	28	28	28	26·2
1·50 „	24	25	24	23	22	24	24	23	22	22	23·6
2·00 „	20	18	19	20	18	21	21	21	20	22	20·0
3·00 „	14	13	14	15	15	...	...	...	...	...	14·2
4·00 „	12	13	12	13	12	...	...	...	...	...	12·4
6·00 „	8	8	$8\frac{1}{2}$	$8\frac{1}{2}$	$9\frac{1}{2}$	...	...	...	...	...	8·5

In the following diagram these results are plotted to scale.



I shall now proceed to compare these results with calculations made from the Law of Fatigue. In the examples of the Law of Fatigue given in No. VII. the work done by the muscles is dynamical work, and consists in lifting weights at a fixed rate until fatigue sets in; but in the present experiments the work done is partly dynamical and partly statical, the latter consisting in the efforts made by the muscles to hold the weight and arm extended in positions varying from the vertical to the horizontal position.

Let W_1, R_1 be the dynamical work and rate of work, and let W_2, R_2 be the statical work and rate of work.

If the work done were purely dynamical or purely statical, we should have, by the Law of Fatigue, either

$$W_1 R_1 = \frac{W_1^2}{T} = \text{constant},$$

or
$$W_2 R_2 = \frac{W_2^{\,2}}{T} = \text{constant},$$
T being the total time of work.

The Law of Fatigue may be applied in one or other of two ways.

1°. The total work done is $W_1 + W_2$, and the rate of work $\frac{W_1 + W_2}{T}$.
Hence the Law of Fatigue gives us
$$\frac{(W_1+W_2)^2}{T} = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The dynamical work is proportional to x, the length of the arm, to $w+\alpha$, the weight plus half the weight of the arm, and to n the number of lifts.

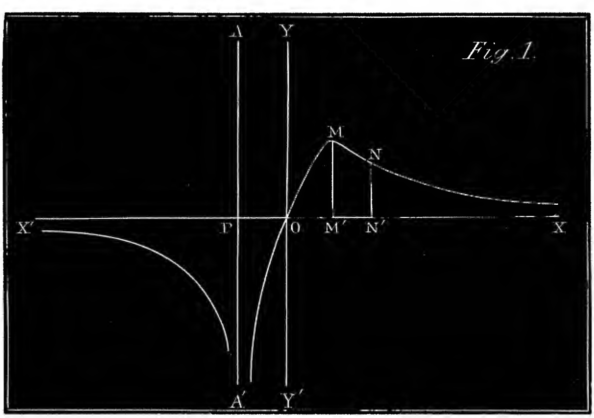
The statical work is proportional to x, the length of the arm, to $w+\alpha$, the weight plus half the weight of the arm, and to T the total time.

The total time T is proportional to

n, the number of lifts,

and to t, the time of each lift.

Hence equation (1) becomes
$$\frac{(w+\alpha)^2 x^2 n^2 (1+\beta t)^2}{nt} = \text{constant},$$
where β is an unknown constant. But $w+\alpha$ and x are constants in these experiments, and hence we find equation (1) reduced to the following:—
$$\frac{n\ (1+\beta t)^2}{t} = \mathrm{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)^*$$



* The general form of this *cuspidal cubic* is shown in fig. 1, in which XX' is the single asymptote, corresponding to
$$t = 0,$$

This represents a *cuspidal cubic*; and we are required to find values for β and A which will satisfy the experiments.

2°. The second method of applying the Law of Fatigue leads to an equation which represents the experiments better than equation (2); and the principle on which it is founded is probably a more correct application of the Law of Fatigue. I assume that fatigue will occur when the dynamical work multiplied by its rate, together with the statical work multiplied by its rate, shall be constant; or if W R represent the total work of all kinds, and *rate of both works*, then

$$W R = W_1 R_1 + W_2 R_2 = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is equivalent to assuming the total rate of work to be

$$R = \frac{W_1 R_1 + W_2 R_2}{W}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

as in the problem of the specific gravity of a binary compound.

Equation (3) becomes at once

$$\frac{W_1^2 + W_2^2}{T} = \text{constant},$$

or

$$\frac{\overline{(w+\alpha)\,x\,n}^2 + \overline{(w+\alpha)\,x\,\beta t\,n}^2}{n\,t} = \text{constant}$$

or finally, since $(w+\alpha)$ and x are constants,

$$\frac{n\,(1+\beta^2\,t^2)}{t} = A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)^*$$

We have now to take equations (2) and (5) in succession, and find which of them corresponds best to the observations. The method I have followed is this:—Let any value of β be assumed, and substituted in equations (2) and (5) for all the values of t; the resulting values of A will differ more or less from each other: let δA be the greatest difference between any two values, and let μ be the number of observations, and $\Sigma . A$ the sum of the values of A; then I determine for each succes-

---

and AA′ is the double asymptote, corresponding to

$$1 + \beta\,t = 0,$$

and having a cusp at negative infinity. There is a hyperbolic branch lying between X′P and A′P.

The portion of the curve with which we are concerned lies between OX and OY.

The number of lifts (n) attains a maximum MM′ when

$$1 - \beta t = 0,$$

and the point of inflexion of the curve N occurs at double the preceding value of t; after which the curve becomes asymptotic to the line OX.

* This equation represents a *central cubic* whose general form is shown in fig. 2 (p. 135). It has a double point at infinity on the axis YY′, which is a conjugate point

sive value of β the quantity $\frac{\delta A}{\Sigma . A}$, and finally choose that value of β which makes $\frac{\delta A}{\Sigma . A} =$ minimum.

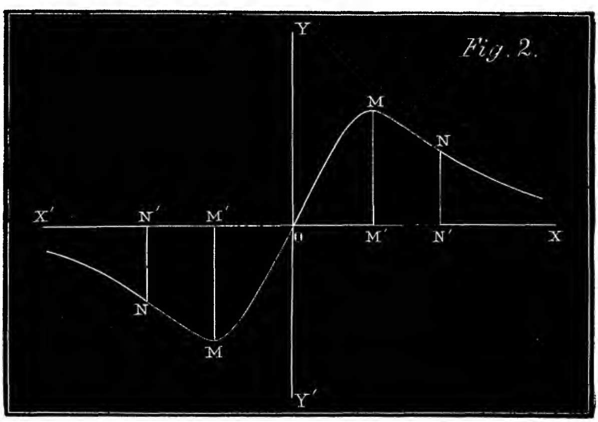
The greatest deviation per cent. of any value of A from the mean value is of course $\frac{\mu \delta A}{2 \Sigma . A} =$ maximum error per cent., where μ is the total number of experiments. Proceeding in this manner, we find, from equation (2), the following—

β.	$\frac{\delta A}{\Sigma . A}$.	
1·0	5·0	
1·4	4·43	
1·5	4·18	
1·6	4·45	
2·0	5·7	(Dr. Macalister.)

This gives a maximum error in the value of A of 16·72 per cent.

Applying the same method to equation (5), we obtain

β.	$\frac{\delta A}{\Sigma . A}$.	
0·6	10·2	
0·9	3·22	
1·0	1·55	
1·1	3·32	
1·6	8·4	(Dr. Macalister.)



(acnode) and not a cusp. The curve is central and has a point of inflexion at the origin, and the axis XX′ is asymptotic on both sides. The tangent at the origin is $n = A\,t$.

The ordinate (n) reaches a maximum for the values

$$\beta\, t \pm 1 = 0,$$

corresponding to M, M′.

The curve has also two real points of inflexion N, N′, corresponding to

$$\beta\, t \pm \sqrt{3} = 0.$$

This gives a maximum error in the value of A of 6·20 per cent. Hence we adopt the equation (5) as the best representation of the observations, and as the best application of the Law of Fatigue.

For $\beta = 1{\cdot}0$, we find

$$
\begin{aligned}
A = {} & 47{\cdot}5 \\
& 53{\cdot}3 \\
& 52{\cdot}4 \\
& 51{\cdot}0 \\
& 50{\cdot}0 \\
& 47{\cdot}0 \\
& 52{\cdot}7 \\
& 51{\cdot}8 \\
\text{Mean} \ldots\ldots & 50{\cdot}7
\end{aligned}
$$

We may now proceed to calculate the values of n from equation (5), using the constants

$$
\begin{aligned}
A &= 50{\cdot}7 \\
\beta &= 1{\cdot}0 ;
\end{aligned}
$$

and thus we obtain

Dr. Macalister.

No.	t.	n (obs.).	n (calc.).	Diff.
1.	0·50 sec.	19·0	20·2	− 1·2
2.	0·66 ,,	24·6	23·4	+ 1·2
3.	1·00 ,,	26·2	25·4	+ 0·8
4.	1·50 ,,	23·6	23·4	+ 0·2
5.	2·00 ,,	20·0	20·2	− 0·2
6.	3·00 ,,	14·2	15·2	− 1·0
7.	4·00 ,,	12·4	11·9	+ 0·5
8.	6·00 ,,	8·5	8·3	+ 0·2

This Table shows a very satisfactory agreement of the observations with the Law of Fatigue expressed by equation (5); and this agreement is also shown in the Diagram on p. 132, where the curve (5) is drawn to scale, and where the individual observations are marked by the small circles.

X. "On Repulsion resulting from Radiation. Influence of the Residual Gas."—(Preliminary Notice.) By William Crookes, F.R.S. &c. Received June 13, 1876.

I have recently been engaged in experiments which are likely to throw much light on some obscure points in the theory of the repulsion resulting from radiation. In these I have been materially assisted by Professor Stokes, both in original suggestions and in the mathematical

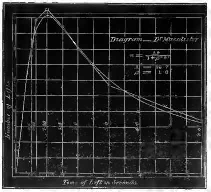


Diagram _ Dr. Macalister
$n = \frac{At}{1+\beta^2 t^2}$
$A = 50 \cdot 7$
$\beta = 1 \cdot 0$
Number of Lifts
1.00
1.5
2.0
3.0
4.0
8.0
Time of Lift in Seconds.

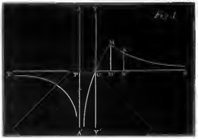


Fig. 1
A
Y
M
N
X'
P
O
M'
N'
X
A'
Y'

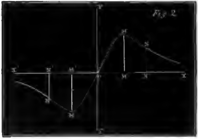


Fig. 2.